Escola Secundária da Sé-Lamego

Ficha de Trabalho de Matemática

Ano Lectivo de 2003/04 — ***Derivadas – 2 (Regras de derivação)*** — **12.º Ano**

Nome: ____	N.º:____	Turma: ___

1. Determine a função derivada de cada uma das seguintes funções, reais de variável real:

a) $f(x) = 3x^2 + 5x\ ;\ D_f = IR$

b) $f(x) = x^3 + x\ ;\ D_f = IR$

c) $f(x) = 3.(x^2 + 2x)\ ;\ D_f = IR$

d) $f(x) = (3x + 2).(x^2 + 1)\ ;\ D_f = IR$

e) $f(x) = (x^2 + 1).(x^3 + 3)\ ;\ D_f = IR$

f) $f(x) = (3x + 1).(x^2 + 1).(x^3 - x)\ ;\ D_f = IR$

g) $f(x) = (x^2 + 2x)^2\ ;\ D_f = IR$

h) $f(x) = (x^2 + 3x)^3\ ;\ D_f = IR$

i) $f(x) = \dfrac{x^3 - 3x}{2}\ ;\ D_f = IR$

j) $f(x) = \dfrac{3}{(x^2 + 2)^5}\ ;\ D_f = IR$

l) $f(x) = \dfrac{x^2 - 2}{x + 1}\ ;\ D_f = IR \setminus \{1\}$

m) $f(x) = \left(\dfrac{x^3 - 1}{x^2 + 1}\right)^3\ ;\ D_f = IR$

n) $f(x) = \sqrt{x + 2}\ ;\ D_f = [-2, +\infty]$

o) $f(x) = \sqrt[3]{x^3 - 1}\ ;\ D_f = IR$

p) $f(x) = \dfrac{1}{(x - 1)^3}\ ;\ D_f = IR \setminus \{1\}$

q) $f(x) = \dfrac{1}{\sqrt{x^2 + 1}}\ ;\ D_f = IR$

r) $f(x) = \left(\dfrac{2x}{x - 1}\right)^3\ ;\ D_f = IR \setminus \{1\}$

s) $f(x) = \sqrt{\dfrac{2x}{x + 3}}\ ;\ D_f =]-\infty, -3[\cup [0, +\infty[$

t) $f(x) = \sqrt{x^4 - 3x}\ ;\ D_f =]-\infty, 0] \cup [\sqrt[3]{3}, +\infty[$

u) $f(x) = \dfrac{2}{(x + 4)^3}\ ;\ D_f = IR \setminus \{-4\}$

v) $f(x) = ((x^3 + 1)(2x^2 - 1))^3\ ;\ D_f = IR$

x) $f(x) = (2 - \sqrt[3]{x})^4\ ;\ D_f = IR$

2. Determine as derivadas das funções reais de variável real, abaixo definidas:

a) $f(x) = e^{x^2}\ ;\ D_f = IR$

b) $f(x) = e^{3x+5}\ ;\ D_f = IR$

c) $f(x) = 2^{x^2 - x}\ ;\ D_f = IR$

d) $f(x) = \ln(2x + 1)\ ;\ D_f =]-\frac{1}{2}, +\infty[$

e) $f(x) = \log_2 (x^2 + 5)\ ;\ D_f = IR$

f) $f(x) = \ln\left(\dfrac{x - 2}{x + 5}\right)\ ;\ D_f =]-\infty, -5[\cup]2, +\infty[$

g) $f(x) = \ln(\sqrt[3]{1 + x^2})\ ;\ D_f = IR$

h) $f(x) = x^2.e^{-2x}\ ;\ D_f = IR$

i) $f(x) = \dfrac{\ln x^2}{e^x}\ ;\ D_f = IR \setminus \{0\}$

j) $f(x) = \ln(e^x - 1)\ ;\ D_f = IR^+$

3. Sobre a definição de derivada e cálculo da derivada pela definição, explore a seguinte ligação:

- http://archives.math.utk.edu/visual.calculus/2/definition.12/index.html

SOLUÇÕES

1.

a) $f'(x) = 6x + 5$

b) $f'(x) = 3x^2 + 1$

c) $f'(x) = 6x + 6$

d) $f'(x) = 9x^2 + 4x + 3$

e) $f'(x) = 5x^4 + 3x^2 + 6x$

f) $f'(x) = 18x^5 + 5x^4 - 6x - 1$

g) $f'(x) = 4x^3 + 12x^2 + 8x$

h) $f'(x) = 3.(x^2 + 3x)^2.(2x + 3)$

i) $f'(x) = \dfrac{3x^2 - 3}{2}$

j) $f'(x) = -\dfrac{30x}{(x^2 + 2)^6}$

l) $f'(x) = \dfrac{x^2 + 2x + 2}{(x + 1)^2}$

m) $f'(x) = \dfrac{(3x^4 + 9x^2 + 6x)(x^3 - 1)}{(x^2 + 1)^4}$

n) $f'(x) = \dfrac{1}{2\sqrt{x + 2}}$

o) $f'(x) = \dfrac{x^2}{\sqrt[3]{(x^3 - 1)^2}}$

p) $f'(x) = -\dfrac{3}{(x - 1)^4}$

q) $f'(x) = -\dfrac{x}{\sqrt{(x^2 + 1)^3}}$

r) $f'(x) = -\dfrac{24x^2}{(x - 1)^4}$

s) $f'(x) = \dfrac{3}{(x + 3)^2} \times \sqrt{\dfrac{x + 3}{2x}}$

t) $f'(x) = \dfrac{4x^3 - 3}{2\sqrt{x^4 - 3x}}$

u) $f'(x) = -\dfrac{6}{(x + 4)^4}$

v) $f'(x) = 3.(x^3 + 1)(2x^2 - 1))^2 \times (10x^4 - 3x^2 + 4x)$

x) $f'(x) = -\dfrac{4 \times (2 - \sqrt[3]{x})^3}{3\sqrt[3]{x^2}}$

2.

a) $f'(x) = 2x.e^{x^2}$

b) $f'(x) = 3.e^{3x+5}$

c) $f'(x) = (2x - 1).2^{x^2 - x}.\ln 2$

d) $f'(x) = \dfrac{2}{2x + 1}$

e) $f'(x) = \dfrac{2x}{(x^2 + 5).\ln 2}$

f) $f'(x) = \dfrac{7}{(x - 2).(x + 5)}$

g) $f'(x) = \dfrac{2x}{3.(1 + x^2)}$

h) $f'(x) = 2x.(1 - x).e^{-2x}$

i) $f'(x) = \dfrac{2 - x.\ln x^2}{x.e^x}$

j) $f'(x) = \dfrac{e^x}{e^x - 1}$

O Professor

Escola Secundária/3 da Sé-Lamego

Ficha de Trabalho de Matemática

Ano Lectivo 2003/04 *Derivadas – 2 (Regras de derivação)* **12.º Ano**

Proposta de Resolução:

1.

a) $f'(x) = (3x^2 + 5x)' = (3x^2)' + (5x)' = 3 \times 2x + 5 = 6x + 5$; $D_{f'} = IR$

b) $f'(x) = (x^3 + x)' = (x^3)' + (x)' = 3x^2 + 1$; $D_{f'} = IR$

c) $f'(x) = (3.(x^2 + 2x))' = 3.(x^2 + 2x)' = 3.(2x + 2) = 6x + 6$; $D_{f'} = IR$

d) $f'(x) = (3x + 2)'.(x^2 + 1) + (3x + 2).(x^2 + 1)' = 3.(x^2 + 1) + 2x.(3x + 2) = 9x^2 + 4x + 3$; $D_{f'} = IR$

e) $f'(x) = ((x^2 + 1).(x^3 + 3))' = 2x.(x^3 + 3) + 3x^2.(x^2 + 1) = 2x^4 + 6x + 3x^4 + 3x^2 = 5x^4 + 3x^2 + 6x$; $D_{f'} = IR$

f) $f'(x) = 3.(x^2 + 1).(x^3 - x) + 2x.(3x + 1).(x^3 - x) + (3x^2 - 1).(3x + 1).(x^2 + 1) = 18x^5 + 5x^4 - 6x - 1$; $D_{f'} = IR$

g) $f'(x) = ((x^2 + 2x)^2)' = 2.(x^2 + 2x).(x^2 + 2x)' = 2.(x^2 + 2x).(2x + 2) = 4x^3 + 12x^2 + 8x$; $D_{f'} = IR$

h) $f'(x) = 3.(x^2 + 3x)^2.(x^2 + 3x)' = 3.(x^2 + 3x)^2.(2x + 3)$; $D_{f'} = IR$

i) $f'(x) = \left(\frac{x^3 - 3x}{2}\right)' = \frac{1}{2} \times (x^3 - 3x)' = \frac{1}{2} \times (3x^2 - 3) = \frac{3x^2 - 3}{2}$; $D_{f'} = IR$

j) $f'(x) = \left(\frac{3}{(x^2 + 2)^5}\right)' = \frac{0 - 3 \times ((x^2 + 2)^5)'}{(x^2 + 2)^{10}} = \frac{-3 \times 5.(x^2 + 2)^4 \times 2x}{(x^2 + 2)^{10}} = -\frac{30x}{(x^2 + 2)^6}$; $D_{f'} = IR$

l) $f'(x) = \left(\frac{x^2 - 2}{x + 1}\right)' = \frac{2x.(x + 1) - 1 \times (x^2 - 2)}{(x + 1)^2} = \frac{2x^2 + 2x - x^2 + 2}{(x + 1)^2} = \frac{x^2 + 2x + 2}{(x + 1)^2}$; $D_{f'} = IR \setminus \{1\}$

m)

$$
\begin{aligned}
f'(x) &= 3.\left(\frac{x^3 - 1}{x^2 + 1}\right)^2 \times \left(\frac{x^3 - 1}{x^2 + 1}\right)' \\
&= 3.\left(\frac{x^3 - 1}{x^2 + 1}\right)^2 \times \frac{3x^2.(x^2 + 1) - 2x.(x^3 - 1)}{(x^2 + 1)^2} \\
&= 3 \times (x^3 - 1) \times \frac{3x^4 + 3x^2 - 2x^4 + 2x}{(x^2 + 1)^4} \\
&= \frac{(3x^4 + 9x^2 + 6x)(x^3 - 1)}{(x^2 + 1)^4}
\end{aligned}
$$

$D_{f'} = IR$

n) $f'(x) = (\sqrt{x + 2})' = \frac{1}{2} \times (x + 2)^{\frac{1}{2} - 1} \times (x + 2)' = \frac{1}{2} \times (x + 2)^{-\frac{1}{2}} \times 1 = \frac{1}{2\sqrt{x + 2}}$; $D_f =]-2, +\infty[$

o) $f'(x) = (\sqrt[3]{x^3 - 1})' = \frac{1}{3} \times (x^3 - 1)^{\frac{1}{3} - 1} \times (x^3 - 1)' = \frac{1}{3} \times (x^3 - 1)^{-\frac{2}{3}} \times 3x^2 = \frac{x^2}{\sqrt[3]{(x^3 - 1)^2}}$; $D_{f'} = IR \setminus \{1\}$

p) $f'(x) = \left(\frac{1}{(x - 1)^3}\right)' = \frac{0 - 3.(x - 1)^2 \times 1}{(x - 1)^6} = -\frac{3}{(x - 1)^4}$; $D_{f'} = IR \setminus \{1\}$

q) $f'(x) = \left(\frac{1}{\sqrt{x^2 + 1}}\right)' = (x^2 + 1)^{-\frac{1}{2}})' = -\frac{1}{2} \times (x^2 + 1)^{-\frac{3}{2}} \times (2x) = -\frac{x}{\sqrt{(x^2 + 1)^3}}$; $D_{f'} = IR$

r) $f'(x)=\left(\left(\frac{2x}{x-1}\right)^3\right)'=3\times\left(\frac{2x}{x-1}\right)^2\times\frac{2.(x-1)-1\times 2x}{(x-1)^2}=-\frac{24x^2}{(x-1)^4}$; $D_{f'}=IR\setminus\{1\}$

s) $f'(x)=(\sqrt{\frac{2x}{x+3}})'=\frac{1}{2}\times(\frac{2x}{x+3})^{-\frac{1}{2}}\times\frac{2\times(x+3)-1\times 2x}{(x+3)^2}=\frac{1}{2}\times(\frac{x+3}{2x})^{\frac{1}{2}}\times\frac{6}{(x+3)^2}=\frac{3}{(x+3)^2}\times\sqrt{\frac{x+3}{2x}}$

$D_{f'}=\left]-\infty,-3\right[\cup\left]0,+\infty\right[$

t) $f'(x)=(\sqrt{x^4-3x})'=\frac{1}{2}\times(x^4-3x)^{-\frac{1}{2}}\times(4x^3-3)=\frac{4x^3-3}{2\sqrt{x^4-3x}}$; $D_{f'}=\left]-\infty,0\right[\cup\left]\sqrt[3]{3},+\infty\right[$

u) $f'(x)=(\frac{2}{(x+4)^3})'=\frac{0-3\times(x+4)^2\times 1\times 2}{(x+4)^6}=-\frac{6}{(x+4)^4}$; $D_{f'}=IR\setminus\{-4\}$

v)

$$\begin{aligned} f'(x) &= (((x^3+1)(2x^2-1))^3)' \\ &= 3.(x^3+1)(2x^2-1))^2\times\left[3x^2.(2x^2-1)+(x^3+1)\times 4x\right] \\ &= 3.(x^3+1)(2x^2-1))^2\times(6x^4-3x^2+4x^4+4x) \\ &= 3.(x^3+1)(2x^2-1))^2\times(10x^4-3x^2+4x) \end{aligned}$$

$D_{f'}=IR$

x) $f'(x)=((2-\sqrt[3]{x})^4)'=4\times(2-\sqrt[3]{x})^3\times(-\frac{1}{3}\times x^{-\frac{2}{3}}\times 1)=-\frac{4\times(2-\sqrt[3]{x})^3}{3\sqrt[3]{x^2}}$; $D_{f'}=IR\setminus\{0\}$

2.

a) $f'(x)=(e^{x^2})'=(x^2)'\times(e^{x^2})=2x.e^{x^2}$; $D_{f'}=IR$

b) $f'(x)=(e^{3x+5})'=(3x+5)'\times(e^{3x+5})=3.e^{3x+5}$; $D_{f'}=IR$

c) $f'(x)=(2^{x^2-x})'=(x^2-x)'\times(2^{x^2-x})\times\ln 2=(2x-1).2^{x^2-x}.\ln 2$; $D_{f'}=IR$

d) $f'(x)=(\ln(2x+1))'=\frac{(2x+1)'}{2x+1}=\frac{2}{2x+1}$; $D_{f'}=\left]-\frac{1}{2},+\infty\right[$

e) $f'(x)=(\log_2(x^2+5))'=\frac{(x^2+5)'}{x^2+5}\times\frac{1}{\ln 2}=\frac{2x}{(x^2+5).\ln 2}$; $D_{f'}=IR$

f) $f'(x)=(\ln(\frac{x-2}{x+5}))'=\frac{(\frac{x-2}{x+5})'}{\frac{x-2}{x+5}}=\frac{\frac{x+5-x+2}{(x+5)^2}}{\frac{x-2}{x+5}}=\frac{7.(x+5)}{(x-2).(x+5)^2}=\frac{7}{(x-2).(x+5)}$; $D_{f'}=\left]-\infty,-5\right[\cup\left]2,+\infty\right[$

g) $f'(x)=(\ln(\sqrt[3]{1+x^2}))'=\frac{(\sqrt[3]{1+x^2})'}{\sqrt[3]{1+x^2}}=\frac{\frac{1}{3}\times(1+x^2)^{-\frac{2}{3}}\times 2x}{\sqrt[3]{1+x^2}}=\frac{2x}{3\times\sqrt[3]{1+x^2}\times\sqrt[3]{(1+x^2)^2}}=\frac{2x}{3.(1+x^2)}$; $D_{f'}=IR$

h) $f'(x)=(x^2.e^{-2x})'=(x^2)'\times(e^{-2x})+x^2\times(e^{-2x})'=2x.e^{-2x}+x^2\times(-2)\times e^{-2x}=2x.(1-x).e^{-2x}$; $D_{f'}=IR$

i) $f'(x)=(\frac{\ln x^2}{e^x})'=\frac{(\ln x^2)'\times e^x-(e^x)'\times\ln x^2}{e^{2x}}=\frac{\frac{2x}{x^2}\times e^x-e^x\times\ln x^2}{e^{2x}}=\frac{2-x.\ln x^2}{x.e^x}$; $D_{f'}=IR\setminus\{0\}$

j) $f'(x)=(\ln(e^x-1))'=\frac{(e^x-1)'}{e^x-1}=\frac{e^x}{e^x-1}$; $D_{f'}=IR^+$

O Professor